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Transient response considerations in a video tape recording system.

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Lieutenant, United States Navy

Submitted in partial fulfillment of
the requirements for the degree of

MASTER OF SCIENCE
IN
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Monterey, California

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ABSTRACT

The transient response limitations of an AM-FM-AM system and in, particular, a video bandwidth tape recording system, are examined and discussed. Reference is made to the applicable existing theoretical work in each of the areas of transient response, namely, response of systems to frequency transients; response of systems to amplitude transients; and techniques for correcting a given response. Further study, and accurate results based on these theoretical concepts, are dependent upon finding an approximation to the system function for use in computation.

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1. Introduction.

The ability to permanently record high-speed electrical data, by which is meant electrical data made available at a rate greater than the human assimilation capability, has always lagged the capacity to produce it. Particularly difficult has been the recording of transient data, many ingenious schemes having been devised to trick it into being recorded, or to circumvent the necessity for recording it. The former method is characterized by photographic techniques utilizing high-speed, rapid-sequence cameras through the medium of sensing devices and the oscilloscope; the latter method is characterized by raw data reduction using complex electronic circuits and computers to categorize results and so reduce the rate of the treated data that it can be handled by conventional, relatively slow-speed recorders. Photography can capture transient phenomena with a great deal of accuracy, but is not suitable for continuous precise recording of random transients; raw data reducion methods yield, in general, only time-accumulated quantizations of the transient data.

A significant breakthrough in the recording art was made in 1956 with the introduction of a television video magnetic tape recorder by the Ampex Corporation. The basic recorder was capable of recording a continuous video signal for slightly more than one hour with a band-pass of nominally fifteen cycles to four megacycles per second. Although this machine was designed for the specific purpose of recording television video, it was apparent that it was suitable, at least in principle, for a much more general application to data recording. Since the original video recorder was not miniaturized in any sense, employed vacuum tubes in its electronics, and possessed certain un-

desirable characteristics from the general data recording point of view, a group was organized in mid-1958 at Ampex Data Products Company to develop a miniaturized data recording version of the machine.[1] Two products resulted from this program in the first few months of 1960: one a basic record-only machine, capable of recording two channels of video data with an essentially flat frequency response from ten cycles to four megacycles per second, plus two auxiliary audio-frequency channels; and the other a rack-mounted equipment with both record and playback functions, using the same basic machine and electronics. The record-only unit is designed to meet Air Force specifications for airborne service and is designated the AN/ALH-4; the rack-mounted record-playback unit is for use on the ground and is designated the AN/GLH-3.

It is the purpose of this paper to examine and discuss the capabilities and limitations of the AN/GLH-3 in pulse recording applications, with specific references to the particular system components or processes which limit the transient response of the entire system. Pulse response criteria will be discussed in a separate section; for the body of the paper, the familiar ones of rise time and overshoot, with reference to the unit step excitation function, will be used.

[1]U. S. Air Force contract AF 33 600-37696 assisting

2. The Ampex Wideband Magnetic Tape Recording and Reproducing System.

A brief summary of the operational principles of the wideband recorder (AN/ALH-4) and recorder/reproducer (AN/GLH-3) is in order before the system characteristics are considered. Since the differences between the two units for the function of recording are negligible---that is, since identical signals into either unit in the record mode should produce the same magnetic pattern on the tape---distinctive reference will not be made to either unit regarding the recording function; moreover, discussion of the reproducing function will be understood to apply only to the AN/GLH-3.

It is assumed that the reader has a passing familiarity with the Ampex television video recording system. The wideband recorder is roughly similar to its predecessor in the recording method and tape transport layout. The mechanics of the tape transport are not of primary interest here; however, a simplified diagram and a photograph of the basic transport are included as Figures 2.1 and 2.2, respectively.

The wideband information is recorded transversely on a two inch width of magnetic tape, subsequent time intervals being recorded on parallel adjacent tracks as the tape is pulled past the rotating head assembly. It will be noted on Figure 2.2 and Figure 2.3, which shows the head assembly separately, that two of the standard head drums are mounted on the head drum motor: when recording two wideband channels, one channel is recorded by each head drum, and the tape is pulled by the heads at twice the speed that it is when recording one channel, so that each head lays down alternate tracks on the tape. Since both channels are identical, only single channel operation will be discussed hereafter.

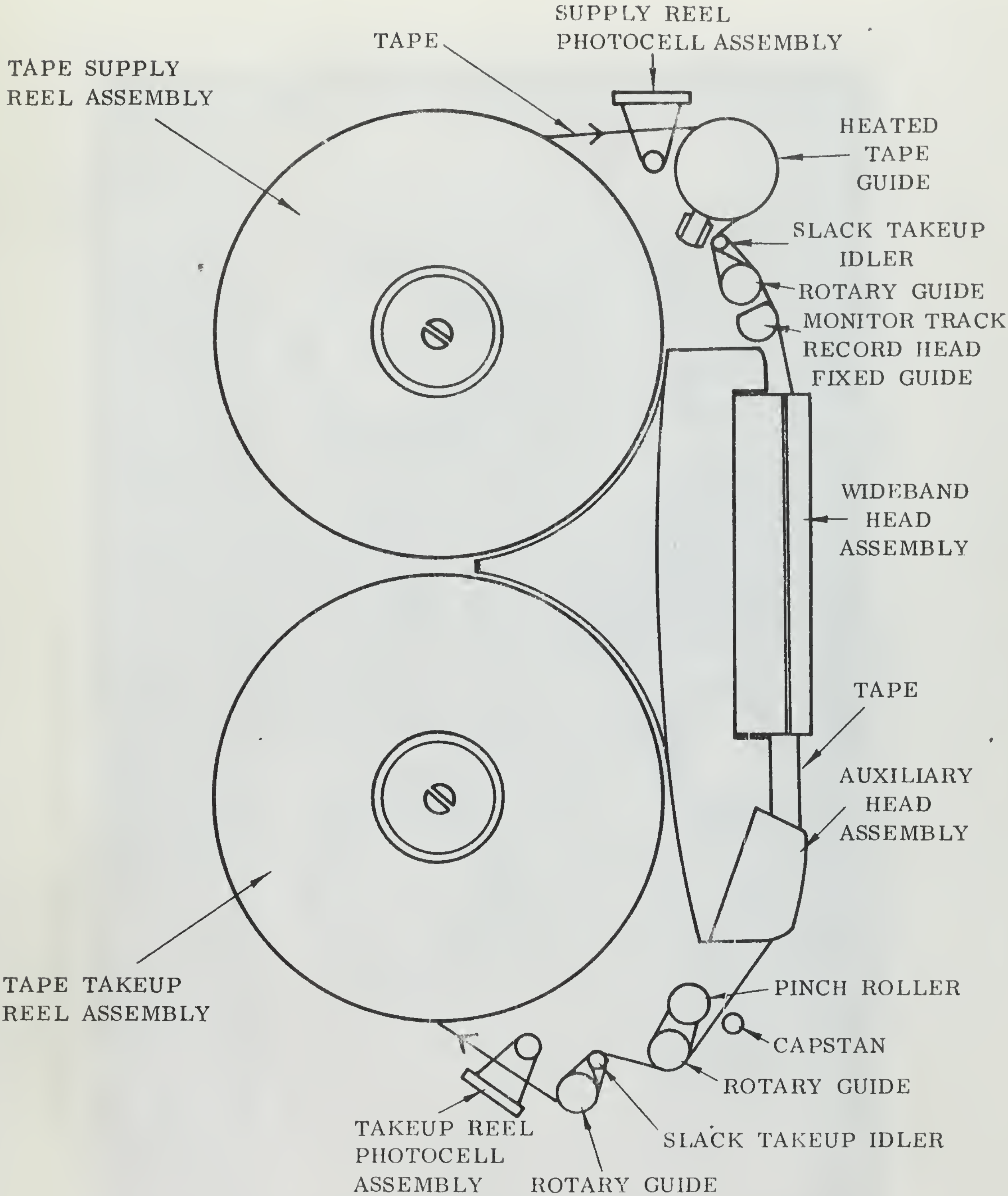


Figure 2.1. Arrangement of Tape Handling Components

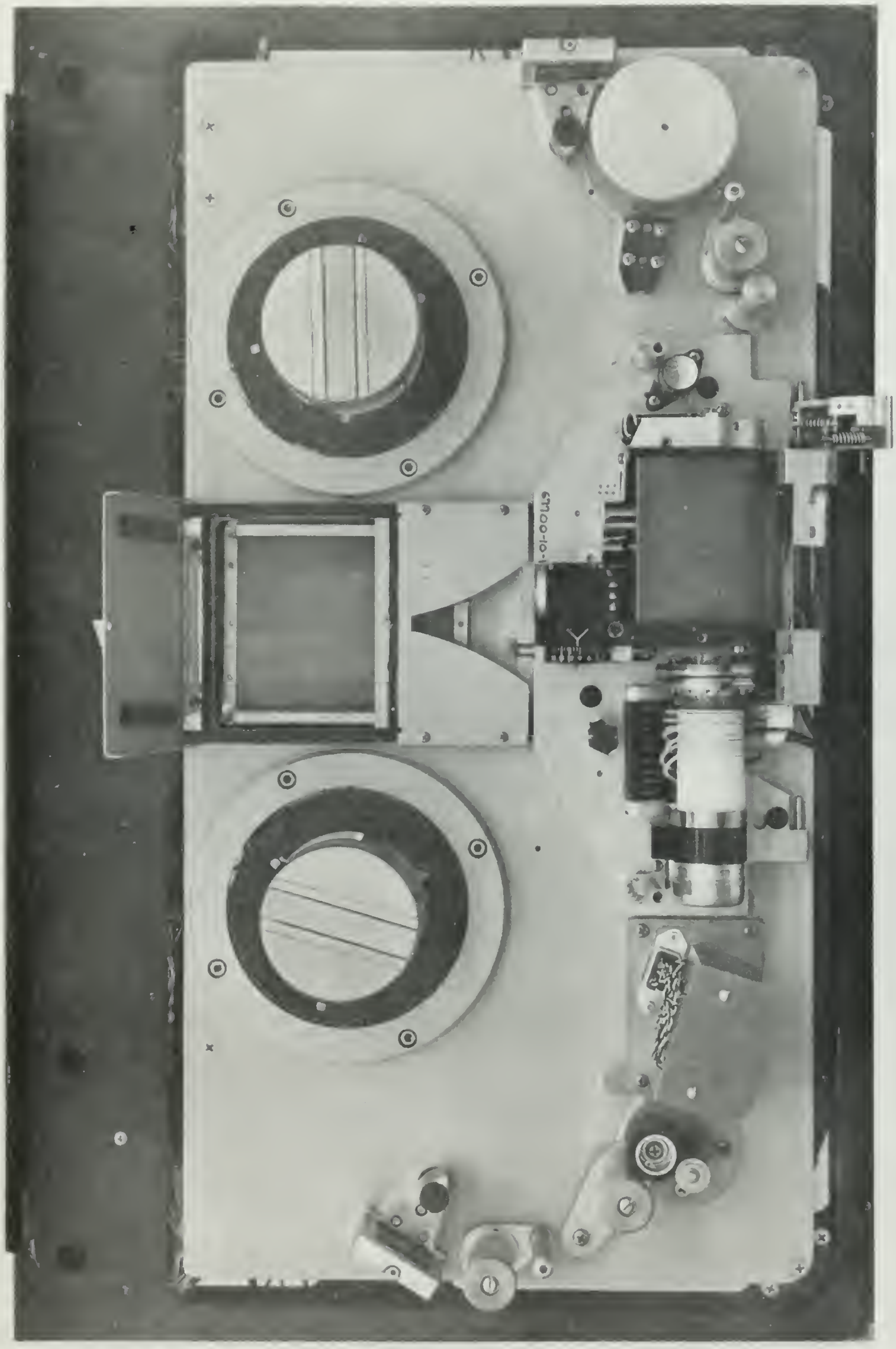


Figure 2.2. Tape Transport. Head Covers Removed

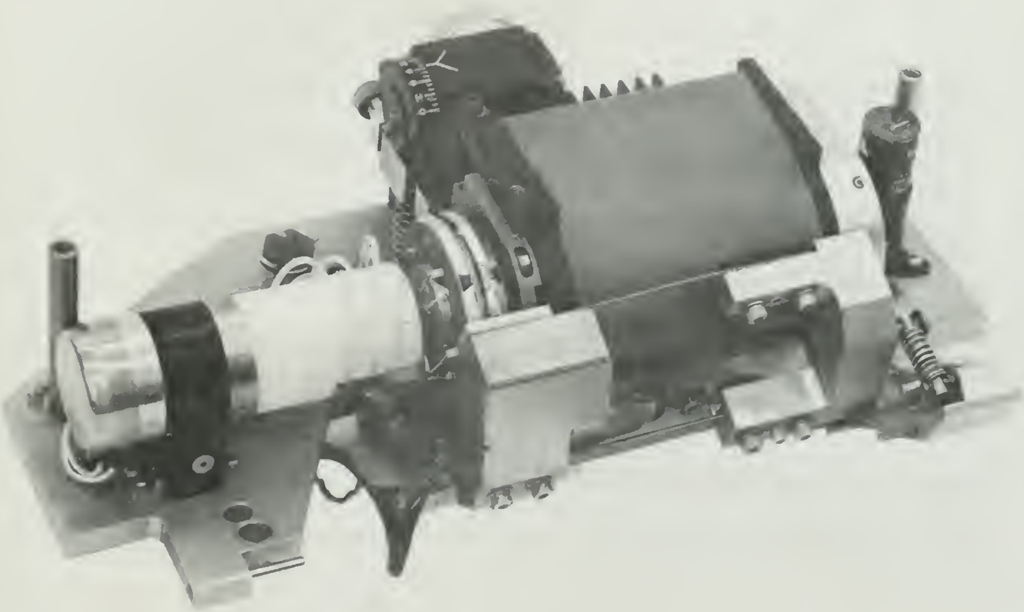


Figure 2.3 . Wideband Rotary Head Assembly

There are four heads spaced at 90° angular intervals about each drum; when recording, all heads are driven continuously, in parallel, by the waveform to be recorded. Since the transverse track length used is 1.78 inches, and the circumferential distance between heads is 1.63 inches, each succeeding head in the rotational sequence contacts the tape and commences recording before the previous one has left the tape: hence, there is an overlap of recorded information on successive tracks of

(track width) minus (head spacing) = 0.15 inches.

This overlap time is utilized upon playback to switch into the playback circuit the head coming onto the tape, and switch out of the playback circuit the head leaving the tape, so that no information is lost. The heads are not left connected in parallel during playback in order to decrease the system noise.

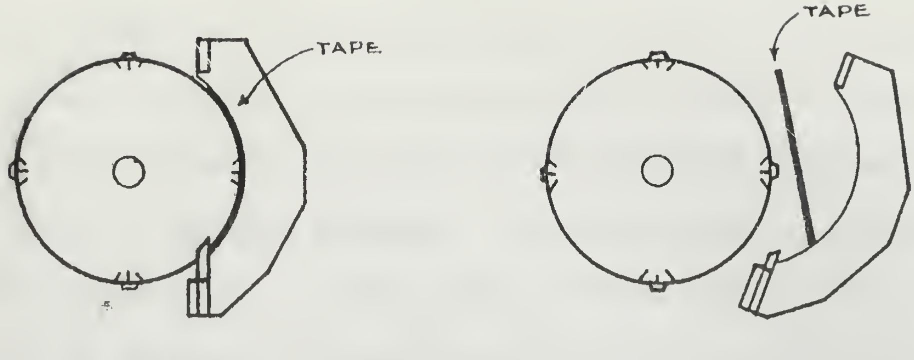
Data is recorded and recovered via frequency modulation. The signal to be recorded is applied to an electrically-variable capacitor circuit, which serves as the tank capacitance of a nominally 36 megacycle, Hartley-type oscillator.

Since the maximum frequency which can be recorded is approximately eight megacycles, the resulting frequency modulation spectrum is heterodyned with the output of a similar but fixed-frequency, 30 megacycle oscillator, and the resulting difference frequencies in the band (roughly) .5 mc to 8 mc are amplified and recorded. When the tape is played back, this signal is amplified, limited and converted into the original amplitude information by a delay line-coincidence type discriminator and low-pass filter.

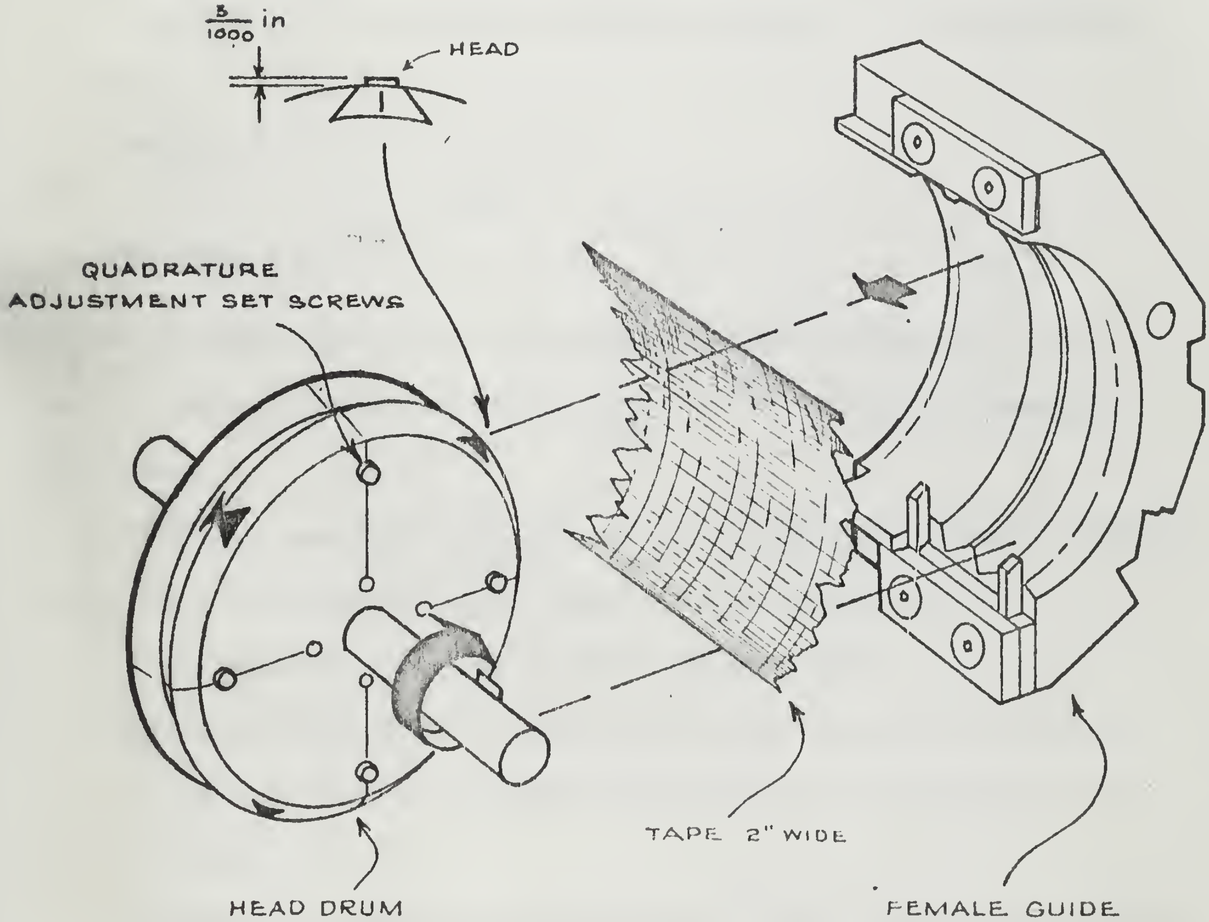
The record-playback time reference of the system is an internal

crystal frequency standard with a designed long-term accuracy of three parts in 10^5. Since mechanical system tolerences and dimensional stability of the tape itself will not permit this accuracy to be preserved on magnetic tape, timing signals are added to the recorded information so that the time base can be corrected in playback as necessary. Specifically, one two-microsecond pulse is recorded by the head entering the top of the tape; another is recorded by the head leaving the bottom of the tape, ten microseconds later. When the tape is reproduced, the time base error is indicated by the time interval between these two reproduced pulses; for example, this interval may be less than ten microseconds if the tape has shrunk, or if the head drum is slightly larger in diameter than that which recorded the tape. Correction is made by effectively stretching the tape in width by forcing the heads deeper into the tape--much as drawing one's finger across a stretched rubber sheet while pressing with considerable force. Figure 2.4 illustrates schematically the relationship between tape, head drum, and female guide; time base correction upon playback is actually made by servo drive of the female guide toward or away from the head drum.

POSITION OF TAPE IN GUIDE



CLOSED OPEN



RELATIONSHIP BETWEEN HEADS, DRUM, TAPE, AND FEMALE GUIDE

FIGURE 2.4

3. Transient Response Limitations of a Low-Pass System.

Before attempting to criticize, improve, or generally evaluate the transient response of an actual band-limited system, it seems proper to try to establish the bounds of the transient response of an "ideal" system of the same bandwidth. It is necessary to appraise the response from the point of view of fixed bandwidth because the system considered has an absolute high frequency limit of operation caused by the head-to-tape recording parameters; the low frequency limit of operation, because of the frequency modulation system, can be considered to be zero.

Propose a problem as follows:

$$\text{Let} \quad H(\omega) = \begin{cases} h(\omega)\, e^{-j\theta(\omega)} & -\omega_c \leq \omega \leq \omega_c \\ \epsilon & |\omega| > \omega_c \end{cases} \qquad \epsilon << 1$$

be the transfer function of a low-pass two-port network, with arbitrary magnitude $h(\omega)$ and arbitrary phase $e^{-j\theta(\omega)}$ within the passband indicated.

Find $h(\omega)$ and $\theta(\omega)$ such that, if a step function voltage is applied to the input terminals, the output voltage displays

a) minimum rise time for a specified overshoot;

b) minimum overshoot for a specified rise time, where overshoot and rise time are defined arbitrarily for the purpose of the problem.

No general solution to this problem is known; however, many papers have been written and networks designed to attempt to approximate solutions for special cases, and experience indicates the general nature of the practical solutions.

Consider the well-known unit step response of the "ideal" low-pass

filter; i.e., the filter with the frequency characteristic

$$H(\omega) = \begin{cases} 1\, e^{-j\omega t_d} & -\omega_c < \omega < \omega_c \\ 0 & |\omega| > \omega_c \end{cases}$$

where t_d is the time delay constant. The response found from the Fourier integral

$$f(t) = \int_{-\infty}^{\infty} F(\omega)\, H(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi} \int_{-\omega_c}^{\omega_c} \frac{e^{j\omega(t-t_d)}}{\epsilon + j\omega}\, d\omega$$

$$= \frac{1}{2} + \frac{1}{\pi}\, si\left[\omega_c (t - t_d)\right]$$

is sketched in Figure 3.1 for two values of ω_c. The fact to be noted is that the maximum slope of the rise, and roughly the rise time, is inversely proportional to the bandwidth, such that

$$\text{Slope}_{max} = \frac{\omega_c}{\pi}$$

and, roughly, $t_{rise} = \frac{\pi}{\omega_c}$.

Note that t_d translates the curve but does not affect its shape.

A well-known special case of modifying the magnitude of the ideal transfer function is shown in Figure 3.2. If the magnitude function is made to droop off as

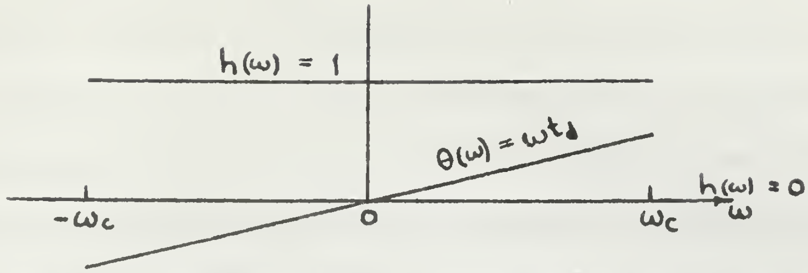
$$e^{-(a\omega)^2}$$

or stating it another way, if

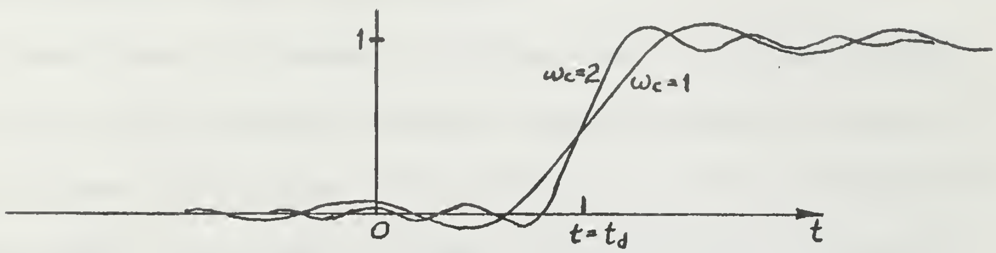
$$\log h(\omega) = -(a\omega)^2$$

such that the response is down one-half at $\pm\omega_c$, then the overshoot and ringing accompanying the previous ideal response are no longer present; however, the maximum slope of the response, and the risetime, remain essentially unchanged.

Having seen two special ideal cases, some small insight may be

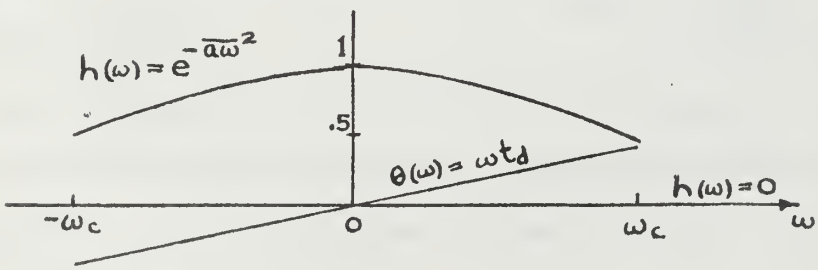


a. FREQUENCY FUNCTION

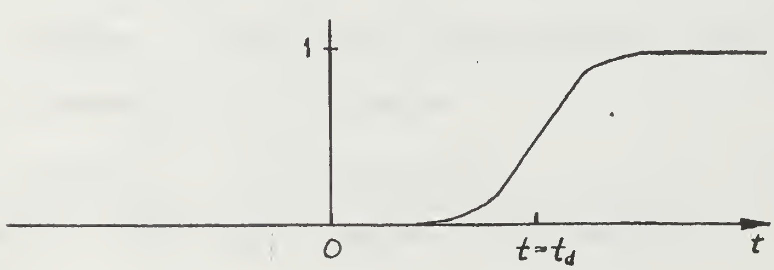


b. STEP RESPONSE

FIGURE 3.1



a. FREQUENCY FUNCTION



b. STEP RESPONSE

FIGURE 3.2

gained into the nature of the problem. First, evaluation of the Fourier integrals to obtain the response for more general cases is nontrivial. Second, transient response depends both on the magnitude and phase characteristic of the L.P. filter. Third, developing a general approach to the problem to, say, minimize rise time with specified bounds on overshoot, would entail a lengthy exercise in the calculus of variations beyond the scope of this paper.

It is the apparent consensus of opinion that it is considerably easier to work with the Laplace transform than the Fourier integral in handling problems of this type. Unfortunately, in gaining computational simplicity, direct contact with the magnitude and phase frequency function is lost.

Consider the two-port network of Figure 3.3 with input voltage and output voltage functions of time.

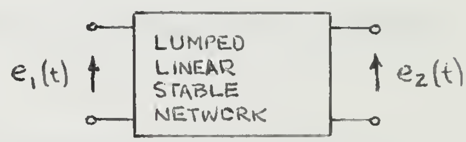


FIGURE 3.3

If $e_1(s) = \mathcal{L}[e_1(t)]$

and $e_2(s) = \mathcal{L}[e_2(t)]$

where $\mathcal{L}$ denotes the Laplace transform, then the transfer function of the system is terms of the complex frequency variable s is

$$T(s) = \frac{e_2(s)}{e_1(s)} \quad .$$

The network possesses a unique time response property which is perhaps best stated conceptually by the equation

$$e_2(t) = \int_{-\infty}^{t} G(t-t')\, e_1(t')\, dt'$$

relating the input and output voltages of the network.

Note that if

$$e_1(t) = \delta(t)$$

the Laplace unit impulse, then

$$e_2(t) = G(t) \quad .$$

Since the network response is unique in both time and frequency, the

functions must be related and, in fact, are the Laplace transforms of one another:

$$T(s) = \mathcal{L}\left[G(t)\right] = \int_0^{\infty} e^{-st} G(t)\, dt$$

$$G(t) = \mathcal{L}^{-1}\left[T(s)\right] = \int_{c-j\infty}^{c+j\infty} \left(\frac{1}{2\pi j}\right) e^{st}\, T(s)\, ds$$

Thus, a more readily applied computational tool may be used in going from the frequency domain to the time domain, or the reverse.

Many interesting comments can be made as a result of manipulation of system transforms. In a series of relatively recent papers,[1] Zemanian develops a set of theorums concerning bounds on the time responses, to the unit impulse and step functions, of various special system functions. These results are not immediately helpful in establishing a guide in the present case, however, because the special natures of the assumed system functions exclude general active network system functions, or because the bounds do not illustrate practical limitations in systems such as this.

Perhaps the best grasp of possible time responses can be obtained by studying the many low-pass filters which have been widely discussed in the literature. The most common of these are the so-called Butterworth and Tchebycheff filters, whose frequency and phase behavior is well-known and available in radio handbooks.[2] The time responses of these practical filters have also been computed;[3] hence, a quick pic-

[1]Armen H. Zemanian, "Bounds existing on the Time and Frequency Responses of Various Types of Networks", PROC I.R.E., May, 1954; 835-839. See also "Further Bounds Existing on the Transient Responses of Various Types of Networks", PROC I.R.E., March 1955; 322-326.

[2]For example, "Reference Data for Radio Engineers", published by International Telephone and Telegraph Corporation, New York.

[3]K. W. Henderson and W. H. Kautz, "Transient Responses of Conventional Filters", TRANS, I.R.E., PGCT, December 1958; 333-347

ture of frequency vs. time response is available which is most instructive. Figure 3.4 indicates the amplitude and phase characteristics, and the response to the unit step function, for each of these low-pass filters with five poles.

Finally, Table 3.1 summarizes the rise times obtained by the ideal and practical low-pass filters considered. The well-known rule of thumb for low-pass filters with about ten percent overshoot,

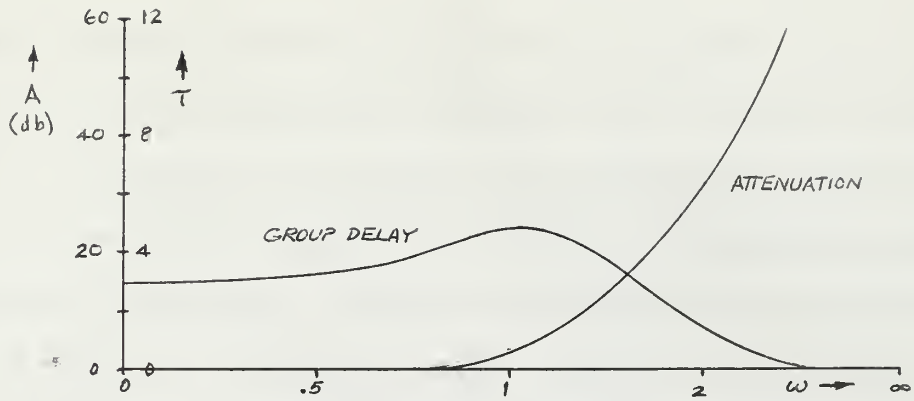
$$t_r \doteq \frac{.45(2\pi)}{\omega_c} ,$$

is also added to the table for comparison.

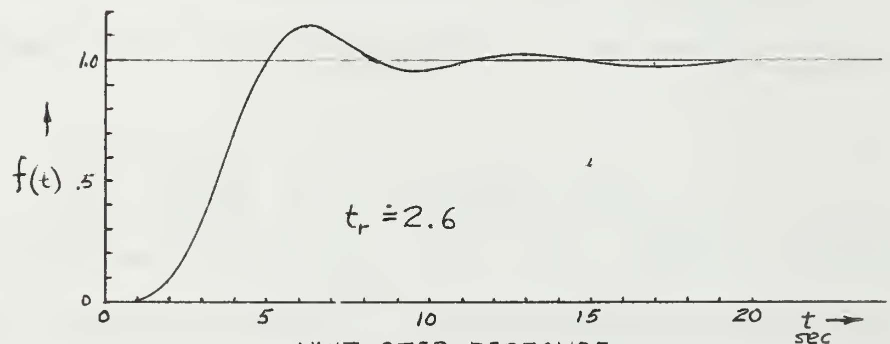
FILTER	RISE TIME (seconds)
Ideal, Flat	3.14
Ideal, Gaussian	3.14
Butterworth ($n = 5$)	2.6
Tchebycheff ($n = 5$, $\rho = .5$)	3.3
Empirical rule-of-thumb	2.5

RISE TIMES FOR UNITY BANDWIDTH L.P. FILTERS

Table 3.1

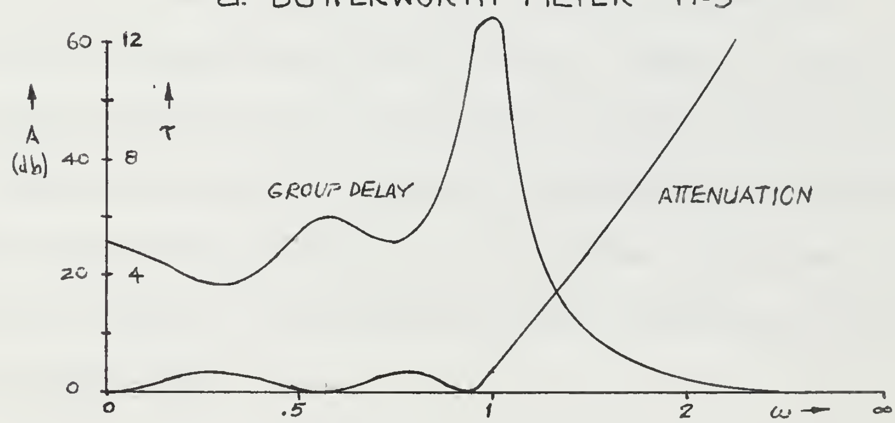


FREQUENCY RESPONSE

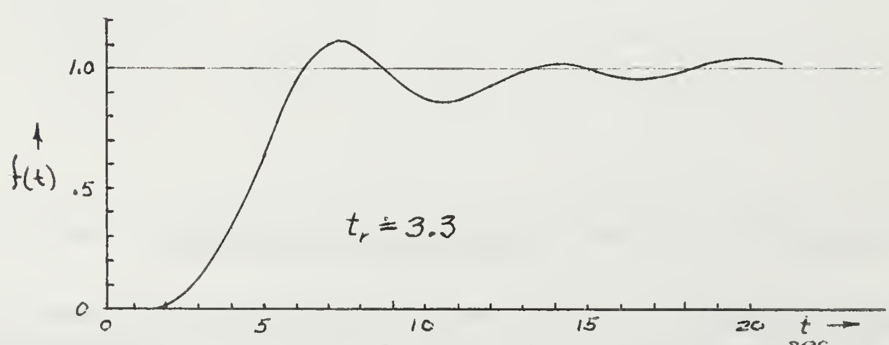


UNIT STEP RESPONSE

a. BUTTERWORTH FILTER n=5



FREQUENCY RESPONSE



UNIT STEP RESPONSE

b. TCHEBYCHEFF FILTER n=5 ρ=.5

FIGURE 3.4

4. Transient Response Limitations of the Wideband Magnetic Tape System.

The tape recording system may be thought of as a low-pass network: an assembly of frequencies is applied to the input terminals and then recovered in altered form, dependent on the overall frequency and phase characteristics of the system. Because the actual system is constructed of lumped elements or equivalent lumped elements, and, to a first approximation at least, can be considered linear, its transfer function can be represented as a ratio of polynomials of the complex frequency variable,[1] as

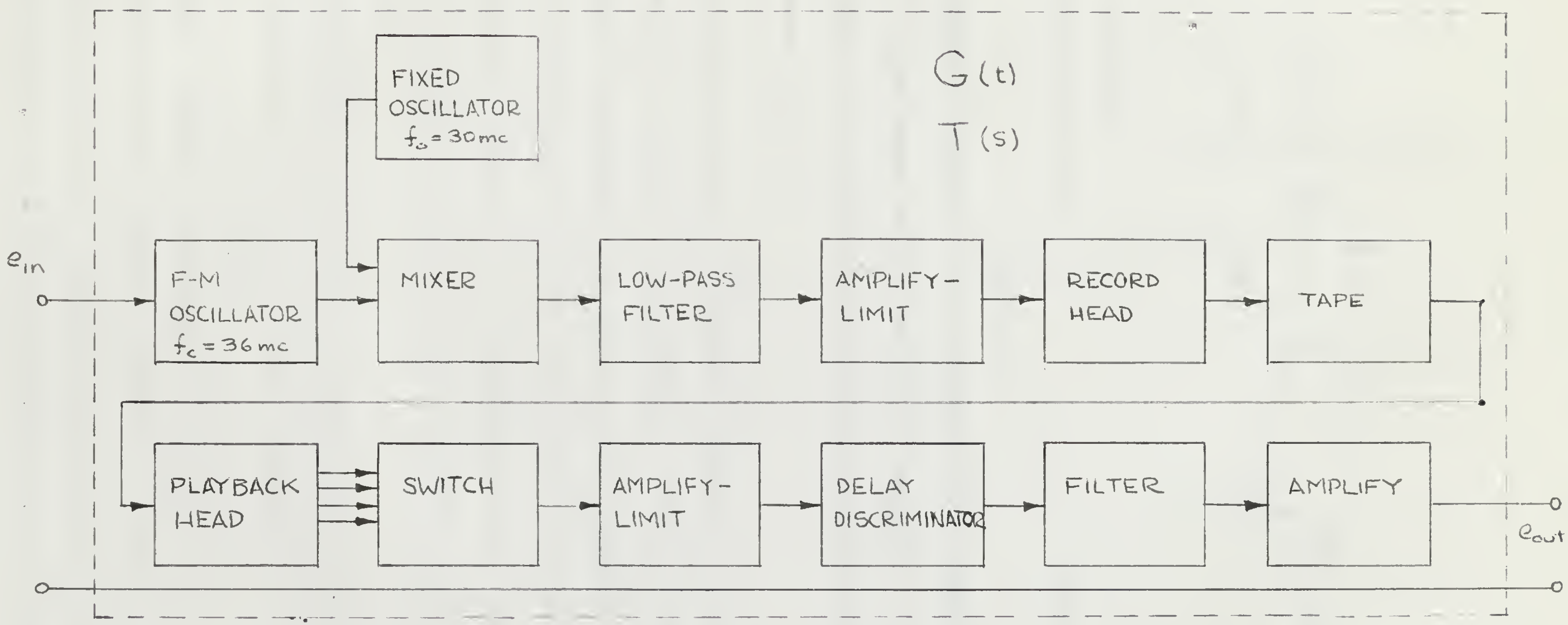
$$T(s) = \frac{N(s)}{D(s)} \quad N, D \text{ polynomials in } S.$$

where T(s) is defined in Section 3 of this paper. Since the product of individual transfer functions of connected networks is the overall transfer function of the group,[2] the system can be considered in broken-down form to simplify the ideas involved. Figure 4.1 is an approximate block diagram of the recording/reproducing circuit of the AN/GLH - 3; this will become the outline for the following discussion.

The frequency modulated oscillator is a modified Hartley with the tank capacitance made up of two voltage-variable (semiconductor) capacitors, as in Figure 4.2a.

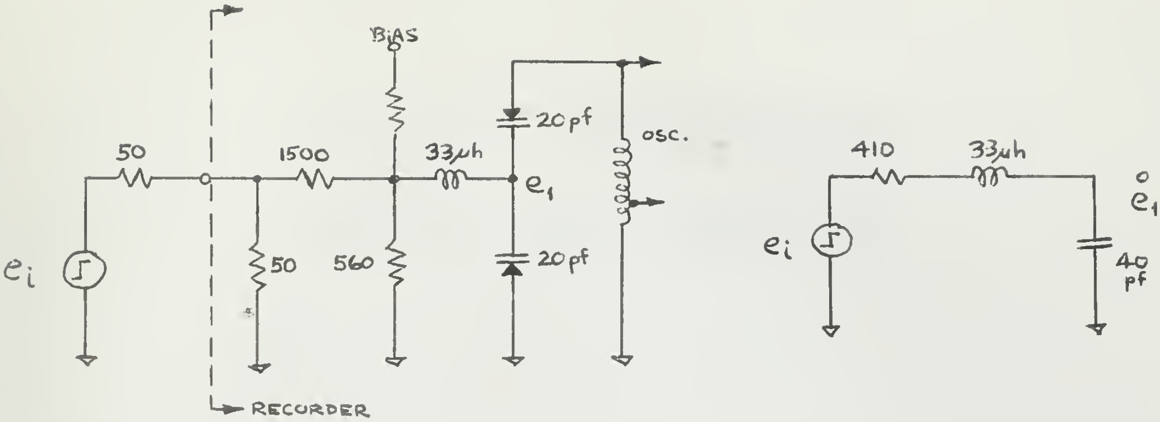
[1]John G. Truxal, Control System Synthesis (New York, 1955), 425.

[2]Only if separability criteria are met. See, for example, Truxal, Control System Synthesis, 89. These criteria are considered to be met in this paper.



SIGNAL PATH THROUGH THE AN/GLH-3

FIGURE 4.1



a. OSCILLATOR INPUT CIRCUIT b. EQUIVALENT

Figure 4.2

The input voltage is applied to the voltage-variable capacitors, thus changing their capacitance and the oscillator frequency. An attempt is made to operate the capacitors at a DC bias such that linear operation is achieved: that is,

$$f_{osc} = k\, e_i \,.$$

Since the oscillator operates at a nominal frequency of 36 megacycles, the tank inductance is about two microhenries. If the reactance of this inductance is considered negligible at four megacycles, with respect to the reactance of the tank capacitors, the input circuit can be reduced to that of Figure 4.2b. The solution of the response of this circuit to a unit step voltage input is trivial, and is plotted in Figure 4.3.

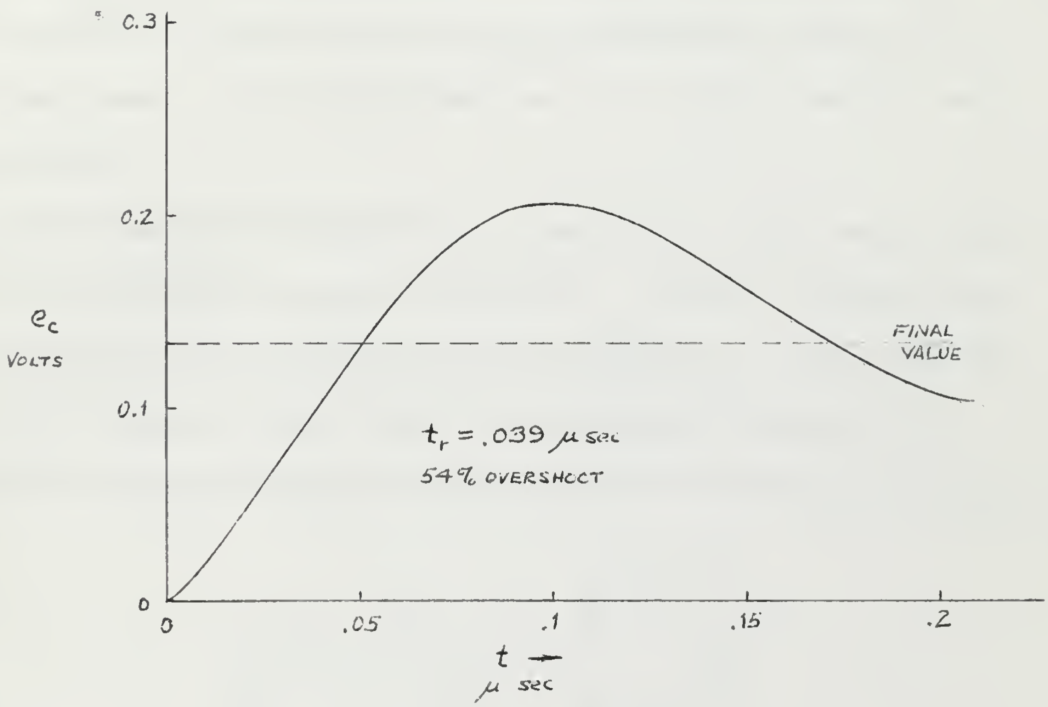
If the rise time is normalized to compare with those given in Section 3, assuming a nominal four megacycle bandwith of the recorder,

$$t_r = 0.98 \,.$$

Thus we see that slightly less than half of the rise time which might be expected for the entire system is required to change the tank capacitance of the f-m oscillator. From the response equation, Figure 4.3, it can be seen that the rise time is governed by the sine term. In the

$$T(s) = 1.023 \times 10^{14} \frac{1}{s\left[(s+6.2\times10^{6})^{2} + 7.18\times10^{14}\right]}$$

$$G(t) = 0.135 + .139\,e^{-6.2t}\sin(26.8t - 1.35)$$



OSCILLATOR INPUT RESPONSE

FIGURE 4.3

response solution, this is

$$\sin \beta t \qquad \beta = \left| \frac{1}{2}\sqrt{\frac{R^2}{L^2} - \frac{4}{LC}} \right|$$

and the rise time is less if the inequality presently obtaining,

$$\frac{4}{C} > \frac{R^2}{L}$$

is strengthened. The most rapid way to do this is to reduce R, which inevitably reduces the damping, causing a longer settling time and greater overshoot. However, notice that the oscillatory term is, in any of these cases, of such a frequency as to be outside the pass band of the recorder.

One might ask why the inductance is used at all, since better rise time with no overshoot is possible without it. Indeed, a trivial computation indicates that with the circuit of Figure 4.4, a rise time of less than one nanosecond may be had, with negligible overshoot; in addition, the amplitude response of this circuit is essentially constant

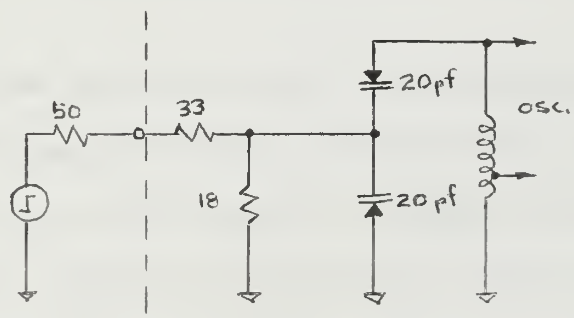


Figure 4.4

over the range of interest. The actual reason for inclusion of the inductance is to so compensate the over-all recorder frequency characteristic that a flat frequency response is obtained.

The action of the frequency modulated oscillator itself is of some interest. No dynamic solution for the time response of such an oscillator is known; furthermore, such a solution is not likely to be simple for two reasons:

a) in general, limited oscillation (i.e., not growing or fading)

is the result of some nonlinear process; and

b) finding the response of circuits with time-varying, and, in particular, step-varying elements, is not trivial.

This problem will not be solved here. It is obvious, however, that the oscillator frequency cannot change instantly, and apparent that the actual rate of change of frequency, under linear assumptions, is dependent on:

a) Q of the oscillator tank,

b) gain of the amplifier, and

c) feedback ratio.

Applying reasoning without attempt at verification, it would seem that the rate of change of frequency would be greatest under conditions of high amplifier gain, and tight coupling of the amplifier to a low-Q tank.

The mixer is a diode square-law device which translates the f-m carrier to six and 66 megacycles; the following LP filter passes the f-m components up to about eight megacycles. Mixing, although utilizing non-linear elements, cannot be considered as a distorting process in its ideal form, because it preserves the amplitude and phase of any input frequency. In actuality, frequency components more than six megacycles below the carrier frequency of the generated f-m wave, are "reflected" from zero frequency when the carrier is heterodyned to six megacycles. Although these out-of-place components contribute to system noise, they have no effect on the response of the system; what does matter is that the lower sideband has been limited to six megacycles, and all lower frequency components have been lost.

The entire system from the mixer output to the discriminator is

an f-m only system. The response of networks to frequency modulated signals has been field of active interest for many years and, contrary to what one might think, articles are still written promulgating new results and arguing old ones. The current state of development of the analysis of system response to f-m excitation is contained in an excellent article by Baghdady[1] which summarizes the earlier work in the field by Carson, Fry, Van der Pol, and Stumpers, and unifies the theory. The philosophy of this work is based on the response of a system or network to an f-m signal being made up of two components: first, that response which can be extrapolated directly from steady-state analysis; and second, an error term which defines the departure of the actual response from that predicted from the steady-state analysis. The criterion to be applied is that the error term must be small for low distortion of the f-m signal, and, in particular,

$$\epsilon_{max} = \frac{1}{2} \left|\frac{d^2\theta}{dt^2}\right|_{max} \left|\frac{Z''(j\omega_i)}{Z(j\omega_i)}\right|_{max} \ll 1$$

where ϵ_{max} is the maximum error, $\theta = \theta(t)$ is the modulating waveform, and $|Z(j\omega_i)|$ is the magnitude of the system function evaluated for $\omega_i(t) = \omega_c + \frac{d\theta}{dt}$. Thus, it is easily seen that the distortion is low if the maximum rate of change of frequency is small with respect to one, or if the magnitude of the network transfer function is flat or linearly related to frequency over the instantaneous frequency range of interest. The bandwidth required to transmit with small distortion is given as

$$BW = K\sqrt{|\theta''(t)|_{max}}, \quad \text{where } K \doteq \sqrt{5\left|\frac{Z''(j\omega)}{Z(j\omega)}\right|_{max}} .$$

[1]Elie J. Baghdady, "Theory of Low-Distortion Reproduction of FM Signals in Linear Systems," TRANS. IRE PGCT, September, 1958; 202-214.

This is somewhat confusing, since K is itself a function of the system transfer function and hence system bandwidth. However, if $Z(j\omega)$ represents a normalized filter, for example, K could be fixed by considering the maximum width of the normalized pass band to be used, and bandwidth found as multiples of the normalized pass band.

Now let us apply this to the system at hand. A few simple calculations show that, for a quasi-step input,

$$\frac{d^2\theta}{dt^2} \doteq \pi\, 10^6 \left(\frac{1}{\text{RISE TIME}}\right)$$ when a deviation of .5 mc is used,

and if the available bandwidth is taken as ten megacycles, then the rise time which can be "faithfully" reproduced is

$$t_r = \frac{K^2\pi}{(BW)^2}\, 10^6 = \frac{K^2}{4\pi}\, 10^{-8}\,.$$

The only convenient information concerning the magnitude of K is in the article by Baghdady, where curves of $\frac{Z''}{Z}$ for the first six orders of the Butterworth filters are given. These represent somewhat ideal cases of a general system, but we shall take for illustration a value of K corresponding to a sixth-order Butterworth over a frequency range slightly in excess of its nominal pass band. Then K = 18, and

$$t_r \doteq .26\,\mu\text{sec}\,.$$

Although these conclusions are interesting, an important point to note is that they specify only the maximum rate of change of frequency which can be passed by a circuit without distortion. This is not transient analysis in the usual sense, although it is certainly a useful philosophy. A more direct approach to the transient response problem is to synthesize an f-m signal which contains a step frequency transition and obtain an expression showing the result of its transmission through a general network. As might be expected, this is a formidable problem; however, it has been attacked and solved, in a

fashion, by both Fourier integral[1] and Laplace transform[2] approaches.

The Fourier integral attack is the same used in Section 3 of this paper: namely, for the network of Figure 4.5, with input voltage $f_1(t)$, output voltage $f_2(t)$,

$$f_2(t) = \int_{-\infty}^{\infty} F_1(\omega)\, H(\omega)\, e^{j\omega t}\, d\omega$$

$$\text{where} \quad F_1(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} f_1(t)\, e^{-j\omega t}\, dt \quad .$$

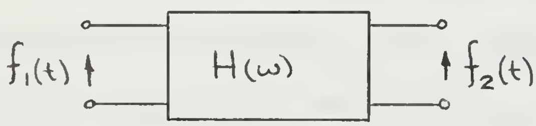


Figure 4.5

In this case,

$$f_1(t) = e^{j\int_{-\infty}^{t} e(\tau)\, d\tau}\; e^{j\omega_0 t}$$

which is the time function depicting an f-m wave with carrier frequency ω_0 and modulation $e(\tau)$. All that remains is to substitute $f_1(t)$ into the integral expression for $F_1(\omega)$, put the resulting $F_1(\omega)$ and the given $H(\omega)$ into the equation for $f_2(t)$, and evaluate the integral. As is usually the case when working with the Fourier integral, difficulties arise because of the non-convergence of certain of the integrals and also because the resulting integrals are difficult to evaluate. The former arises in the course of development[3] of the final integral expression, and is successfully obviated by Gumowski; the latter remains as a challenge to any potential user. The final equation gives the output, for a step

[1]Igor Gumowski, "Transient Response in FM", PROC IRE, May, 1954; 819-822.

[2]R. E. McCoy, "FM Transient Response of Band-Pass Circuits", PROC IRE, March 1954; 574-579.

[3]Gumowski, "Transient Response", 819.

frequency input, as a function of the time response to the impulse function:

$$f_2(t) = e^{j\omega_o t}\left[H(o) + e^{jat}\int_{-\infty}^{t} e^{-ja\tau} h(\tau)\,d\tau - \int_{-\infty}^{t} h(\tau)\,d\tau\right]$$

where $h(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{j\omega t} H(\omega)\,d\omega$,

$H(o)$ is the amplitude function of the network evaluated at ω_o,

a is the "frequency sweep factor" .

A simple parallel RLC network is evaluated on this basis, and curves are drawn for the resulting response for various values of the "sweep index"

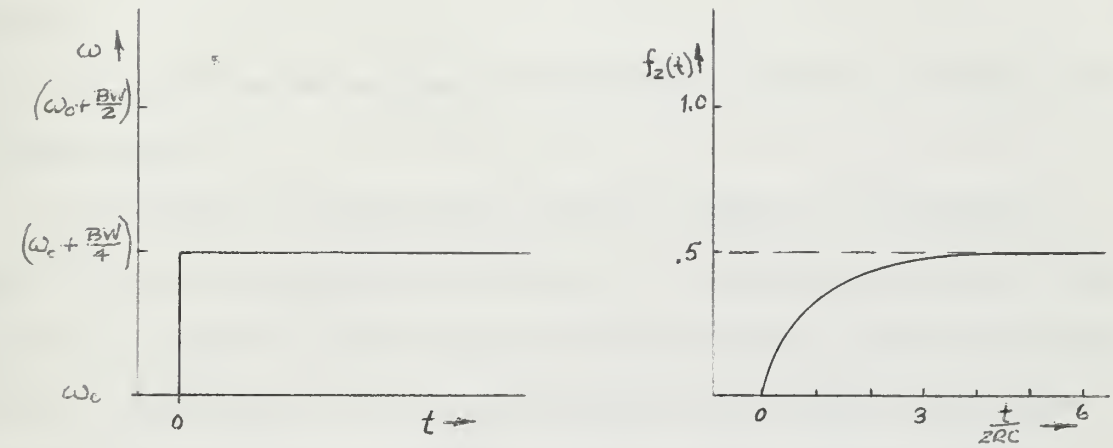
$$m = \frac{2(\text{percent of circuit bandwidth swept})}{(\text{Circuit 3db bandwidth})}$$

when the parallel circuit is resonant at the f-m carrier frequency. This action is illustrated in Figure 4.6, where the time responses to the right of the frequency step graphs are those obtained from the output of a perfect discriminator acting on the function $f_2(t)$.

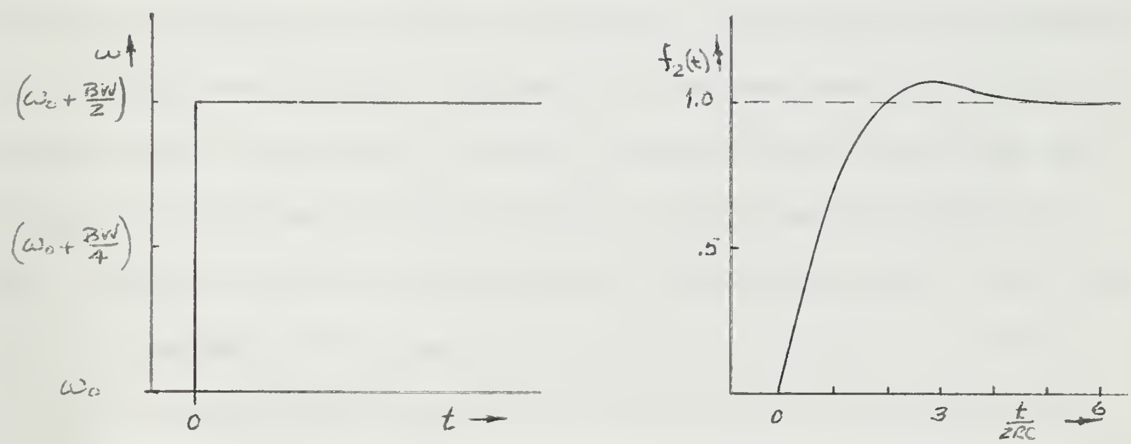
The Laplace transform approach[1] is straightforward and, with one small exception, follows the classic book of Gardner and Barnes[2] in its entirety. In principle, the carrier frequency is applied to a network, as Figure 4.5, until time t = o, and then switched off; the resulting transient decay is found. Also, at t = o, a new frequency is applied to the network, and the transient buildup is found. The sum of the resulting transients is the system response to the (equivalent) step frequency input. The same parallel circuit is analyzed by

[1]McCoy, "FM Transient Response", 574.

[2]M. F. Gardner and J. L. Barnes, Transients in Linear Systems (New York, 1942), I.



a. FREQUENCY STEP = $\frac{BW}{4}$



b. FREQUENCY STEP = $\frac{BW}{2}$

FREQUENCY STEP — TIME RESPONSE PAIRS

FIGURE 4.6

McCoy, and the results are essentially the same; however, additional cases are considered where the

a. Frequency step ends at the circuit resonant frequency, and

b. Frequency step is centered about the circuit resonant frequency.

The results of the two analyses are apparently identical in their overlapping areas.

Now consider the application of these results to the system at hand. First, no quantitative information is immediately available from either one, since the system function must be known to start the computation. The repetition of their development and results has not been without cause, however; in general, it may be said that the change of frequency with respect to time at the output terminals of a two-port, when a step change of frequency is applied to the input, has characteristics similar to the time response of a low-pass network to the unit step function. Certainly, the actual response time is less than that computed for Baghdady's quasi-stationary case, when the output frequency must follow the input frequency to some small error. Also, the work of Gumowski or McCoy may be made, with some effort, to fit the case of the record heads, which are resonant with the various stray capacities at about 6.5 megacycles; with a half-megacycle deviation of the six megacycle carrier, the case of a step frequency ending at circuit resonant frequency is applicable. Unfortunately, the works cited do not immediately apply because a high-Q assumption has been made in both cases.

The delay-coincidence discriminator may be considered to be in the f-m system, where previous analysis applies, until the final output

is realized. If infinite limiting is achieved prior to the discriminator coincidence gate, the output of the discriminator will be a train of rectangular pulses with a period equal to one-half the reciprocal of the input frequency, and of constant width. Thus the pulses would have a minimum 11 megacycle rate, and the change in DC level brought about by the changing frequency would have a maximum delay of .091 microseconds. Although the actual input to the discriminator coincidence gates is not a square wave, but instead is a rough sine wave, the transient considerations are unchanged.

The last link in the chain, then, is the low-pass filter at the discriminator output. Here we are back on familiar ground, and the literature is rich with material to assist in obtaining the required compromise between rise time, overshoot, settling time, and cutoff characteristics. Since discriminator coincidence gate imbalance results in a component of the discriminator output at f-m frequency, the output filter must have large attenuation at 5.5 megacycles, say 40 db. If the transient characteristics of the various filters are examined[1], one might gain the impression that the "Bessel" filter was the answer to the transient problem. However, as might be expected, there is nothing magic about the pole locations for the Bessel filter; the transient response comes with an extremely gradual attenuation in the stop band. To examine the properties more closely, it is seen from the normalized bandwidth plot of the magnitude of the filter attenuation[2] that, with n = 9, 40 db of attenuation is available at

[1]Henderson and Kautz, "Transient Response of Conventional Filters", 342.

[2]W. E. Thomson, "Networks With Maximally-Flat Delay", Wireless Engineer, October, 1952; 257.

$\omega = 10.5$. If this attenuation is made to occur at 5.5 mc, a filter with $\omega = 2\pi\ .523 \times 10^6$ would be required, which would then have a rise time of .2 microseconds. This is somewhat slow when compared to that of more conventional filters. The characteristics of various seventh-order filters for this application are listed in Table 4.1; each has an attenuation of 40 db at 5.5 megacycles. The Cauer[1] filters appear attractive from the bandwidth point of view, but their transient response has not been computed. Their group delay approximates that of a similar Tchebycheff filter except in the neighborhood of $\omega = \omega_o$, where the group delay of the Cauer filter tends to infinity.

LP FILTER		3db BANDWIDTH (mc)	RISE TIME (sec)	OVERSHOOT (%)
Butterworth		2.82	.158	14
Tchebycheff	$\rho = .17$	4.23	.124	12
Bessel		1.65	.208	2
Cauer[2]	$\rho = .20$	5.02	-	-

Table 4.1

The compromise made when selecting a filter should be determined by the application. If the primary use of a given equipment is pulse recording, transient response should be the main concern, and a filter which sacrifices a flat bandpass for less overshoot, for example, may be indicated. For general video use where accurate pulse shapes need not be preserved, one of the flat, sharp cutoff filters, which yield a wide bandwidth, may be preferable. If the application is pulse recording, a further decision must be made on the relative importance

[1] R. Saal and E. Ulbrich, "On the Design of Filters by Synthesis", TRANS IRE PGCT, December, 1958; 315.

[2] ibid., 323. $\theta = 77$.

of rise time and overshoot. In general, two criteria apply:

a) The accuracy with which pulses can be located in time depends upon their rise time;

b) Overshoot and settling time are of primary importance if accurate amplitudes of closely spaced pulses are required.

5. Correction of Transient Response.

Since the transfer function of any system is the product of the transfer functions of its separable components, it is reasonable to assume that a network external to the system may be added whose transfer function, when multiplied with that of the system, results in an overall transfer function which has a better transient response. It is a well-known fact that if the group delay of a network is made more constant the transient response improves, and phase correcting networks have been used with success in many systems.[1] However, the transient correction obtained is not optimal if only the phase is corrected. This is made obvious by the following development:[2]

Let $f_2(t)$ be the actual transient response of the system;

$r(t)$ be the desired response.

$F_2(w)$ is the Fourier transform of $f_2(t)$.

$R(w)$ is the Fourier transform of $r(t)$.

Then, by the multiplication property,

$$F_2(\omega)C(\omega) = R(\omega) \quad ; \qquad C(\omega) = |C(\omega)| e^{-j\phi(\omega)}$$

where $C(\omega)$ is the Fourier transform of a correcting network;

$|C(\omega)|$ is the necessary correction to the magnitude of the given transfer function;

$e^{-i\phi(\omega)}$ is the necessary correction to the phase of the given transfer function.

The question of realizability now arises: namely, is it possible

[1] G. L. Fredendall, "Delay Equalization in Color Television", PROC IRE, January, 1954; 258-262.

[2] John C. Pinson, "Transient Correction of Means of All-Pass Networks", Technical Report 324 of the Research Laboratory of Electronics, Massachusetts Institute of Technology, Cambridge, Mass., dated May 13, 1957.

to synthesize a network with the transfer function $C(\omega)$? The answer, of course, depends on the nature of $C(\omega)$. If the inverse transform of $C(\omega)$

$$c(t) = \int_{-\infty}^{\infty} C(\omega)\, e^{j\omega t}\, d\omega$$

is identically zero for all negative time and bounded for all positive time, then $C(\omega)$ is realizable. The fact that a realizable network exists which will correct the transient response of a given system is not necessarily a cause for rejoicing, however, because the actual realization in lumped R, C, and L may not be feasible from an engineering standpoint. Furthermore, the usual procedure is to obtain the desired magnitude of the system transfer function by manipulation of system constants, based on considerations other than transient response, and then adjust only the phase to obtain the desired transient characteristic. There is at least one good reason for this procedure, and that is simplicity. Fortunately, there exists a class of networks, called all-pass networks, or constant-resistance lattices, whose transfer functions have a constant magnitude but a controllable (within limits) argument. Thus, using the all-pass network, we attempt to synthesize

$$e^{-j\phi(\omega)}$$

or, if this is not possible, to approximate this phase function in some sense. When the best possible correction has been made, consider the resulting response to be $r'(t)$

so that

$$R'(\omega) = K F_2(\omega)\; e^{-j\,\theta(\omega)}$$

where K is an arbitrary gain constant and $\theta(\omega)$ is the realized corrective phase function.

The error in response is then

$$e(t) = r(t) - r'(t) \; .$$

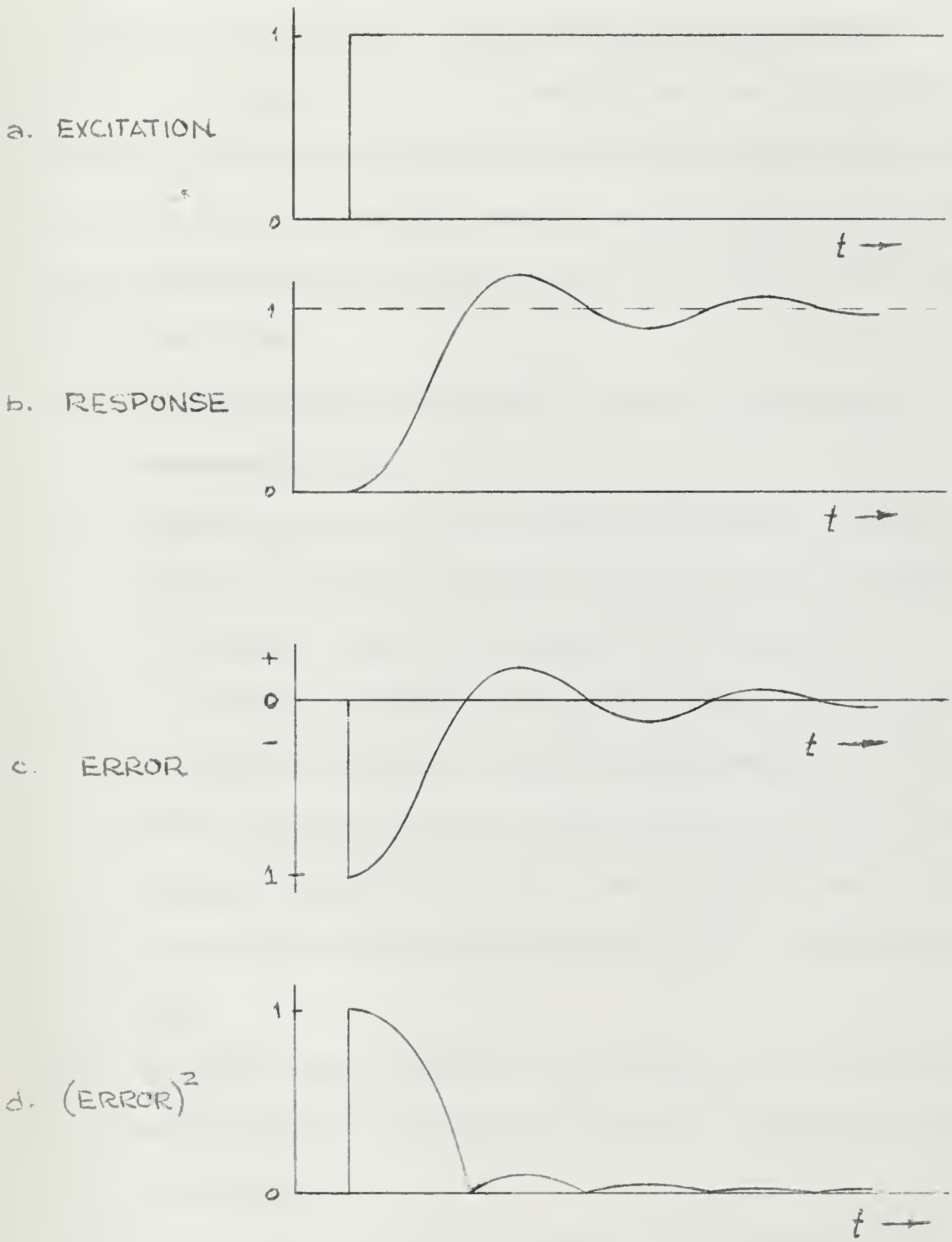
Now consider the problem of minimizing e(t), the error in transient response, and, in particular, what is meant by minimizing the error. Apply a step function voltage to a network, whose response is depicted in Figure 5.1b. Neglecting time delay, the resulting error of response, compared to the input function, is shown in Figure 5.1c. In minimizing this error, we may choose to minimize it for all time, in which case we could minimize the area lying between the error curve and the axis; or, what is equivalent but easier to state mathematically, minimize the square of the error for all time. Then we would minimize

$$\int_{-\infty}^{\infty} e(t)^2 \, dt$$

which is the area under the $e(t)^2$ curve, Figure 5.1d. This problem can be solved, by the calculus of variations[1], to yield the appropriate all-pass network phase function. However, this type of minimization does not necessarily yield the best rise time for the simple reason that errors at all times have been weighted equally in determining the correction, whereas a better rise time would result if the error near t = o were minimized, to the detriment of the error with t large. Again, if settling error were important, it may be desireable to weight the error more heavily at some time greater than zero. Thus a weighted error function could be used, and the calculus of variations again applied to find the optimum $\theta(w)$. For the simple problem used as an example by Pinson, the improvement was slight in any case, although there is no reason to believe that the same would apply to more complicated networks with more distorted phase functions.[2]

[1] Pinson, "Transient Correction", Chapter 4.

[2] For a broad discussion of realization techniques for all-pass networks, see James E. Storer, Passive Network Synthesis (New York, 1957), Chapter 17.



STEP RESPONSE ERRORS

FIGURE 5.1

6. Conclusions, and Recommendations for Further Study.

Although this study was undertaken to examine the limitations of the AN/GLH-3 recorder, most of the information and discussion is applicable to any system in which transient response is important. As applied to the AN/GLH-3 in particular, the final value of any modification can be determined only by controlled experiment, but the following general statements may be made, based on the text of this paper and the references cited:

a) The input circuit, as coupled to the f-m oscillator, is unnecessarily slow.

b) Resonance effects in the frequency modulation circuitry should be avoided in the swept frequency range altogether, and damped as heavily as possible if unavoidable.

c) For accurate reproduction of pulse shapes, the maximum rate of change of frequency in an f-m system must be held to a readily calculable maximum; therefore, the less the total frequency change for any given transient, the better the response time of the network will be, for a given distortion.

d) The filter used to separate the signal from the f-m carrier must be chosen on the basis of the use to which the machine is to be put.

e) All-pass phase correcting networks will improve system response; their use is dictated by their engineering feasibility.

It is to be noted that quantitative computations must be based on the actual transfer function of the system; thus, if real use is

to be made of the concepts expressed in this paper, the system transfer characteristics must be approximated by an appropriate mathematical function.[1,2] Further study should be directed toward such an evaluation of the system function and the carrying through of the calculations in the areas mentioned. It is felt that the most fruitful effort would be in the area of corrective all-pass networks for the system with a wide-band output filter, such as the Cauer.

[1]F. BaHli, "A General Method of Time Domain Synthesis", TRANS IRE, PGCT, September, 1954; 21

[2]Storer, Passive Network Synthesis, 303-304.

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